EFFICIENT DAY-TO-DAY SIMULATION OF TRAFFIC SYSTEMS WITH APPLICATIONS TO THE EFFECTS OF PRE-TRIP INFORMATION

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ABSTRACT

Markov traffic assignment models are attractive tools for representing the day-to-day evolution of traffic flows over road networks. They are simpler than microscopic ('car-following') simulation models, but incorporate considerable flexibility in the manner in which traveller route choice is related to past experience and pre-trip information. In this paper we describe our own Markov traffic assignment model, and its implementation in a new software package MARTS (*M*arkov *A*ssignment for *R*oad *T*raffic *S*ystems) that we are developing. We illustrate the capabilities of MARTS for analyzing the effects of pre-trip information using an example based on the road network in a region of the U.K. city of Leicester. Simulation of this system under various scenarios suggests that provision of high quality pre-trip information can have unexpected results if travellers react in a very volatile fashion.

INTRODUCTION

Early equilibrium assignment models (such as Wardrop's user equilibrium (1)) defined link flow patterns that could be written as the solution to some relatively tractable optimisation problem. However, these models had deficiencies which are now widely recognised. In particular, the lack of explicit representation of traveller learning (i.e. how a traveller's route choice depends upon past experience) in equilibrium assignment models makes them very crude tools for assessing the impact of some types of ITS on a network. See (2), (3) and (4) for further comments. Recent developments in traffic assignment have focused closely on traveller learning as a (typically stochastic) dynamical process, but derivation of the macroscopic properties (e.g. link flows) of such models leads to intractable mathematics. As a result, simulation has become a widely used tool for learning about the evolution of traffic flows in modern traffic assignment models.

Modern assignment models typically fall into one of two classes: within day dynamical models, or day-to-day dynamical models. The former type of model is usually the more detailed, and is often defined at a highly microscopic level (e.g. 'car-following' simulation models). The latter class contains many models that can usefully be described as mesoscopic, in that day-to-day traveller learning is represented in a detailed fashion, but the daily traffic assignment does not allow for within-trip decision making. These models include Cascetta's (5) increasingly popular Markov chain assignment models. (See also (6) and (7).) For investigation of many aspects of ITS (e.g. in-vehicle guidance systems) it is natural to employ within-day models. However, at other times (when interest is in the effect of pre-trip information, for example), both within-day and day-to-day models are possible tools. In such circumstances it is important to recognise the additional complexity of within-day models is not an advantage in itself. Indeed, often a simpler 'broad brush' approach to modelling provides far more reliable results than an attempt to model too much intricate detail. Furthermore, even with today's fast computers, simulation of the medium or long-term evolution of traffic flow over even a moderately sized network can be computationally infeasible for within-day models, but is potentially manageable for day-to-day models.

In this paper we describe the development of a new software tool, MARTS, for implementing day-to-day traffic assignment using the Markov chain framework of Cascetta (5). We outline some new ideas for markedly increasing the simulation speed of such an assignment process with probit route choice model. This increased efficiency means that our software can feasibly run long simulations, and hence assess the behaviour of a modelled transport system over the medium-term. This allows us to assess the impact of pre-trip information on a system at a number of time frames. Our work is illustrated using the road network from a region of the UK city of Leicester.

METHODOLOGY

MARKOV ASSIGNMENT MODELS

We develop a traffic assignment model based on Cascetta's (5) Markov chain framework as follows. Consider the evolution of link and route flows on a network during a particular observational period (e.g. morning peak-hour) over a sequence of (week)days. Let $\mathbf{x}(t)$ and $\mathbf{y}(t)$ be the vectors of link and route flows respectively on day t. These vectors are related by

$$\mathbf{x}(t) = A\mathbf{y}(t)$$

where A is the standard link-path incidence matrix. Let $\mathbf{k}(t) \equiv \mathbf{k}(\mathbf{x}(t))$ be the vector of *measured* link costs generated by the flows on day t, and the $\mathbf{c}(t)$ be the corresponding measured route costs.

In our model, travellers on day t are randomly divided into two classes. A proportion α of these travellers are *habitual* while the remainder are *selective*. The habitual travellers use the route that they took on the previous day (i.e. day $(t-1)$). Each selective traveller reconsiders his/her route choice on day t based upon experience of travel costs on the previous m days (where m can be interpreted as effective memory length). To represent heterogeneity amongst selective travellers, each of them samples a route for day t from the (conditional) route choice probability distribution $\pi(t)$, which is defined in terms of the measured route costs over the previous m days. The link flow pattern for day t is then given by

$$\mathbf{x}(t) = \alpha\mathbf{x}(t-1) + \widehat{\mathbf{x}}(t) \tag{1}$$

where $\widehat{\mathbf{x}}(t) = A\widehat{\mathbf{y}}(t)$ is the new vector of link flows generated by sampled route choices ($\widehat{\mathbf{y}}(t)$) of the selective class of travellers. More specifically, if $\widehat{\mathbf{y}}_i(t)$ is a sub-vector of $\widehat{\mathbf{y}}(t)$ comprising only those routes corresponding to the ith origin-destination (OD) pair, then

$$\widehat{\mathbf{y}}_i(t) \sim \text{multinomial}\left((1-\alpha)n_i, \pi_i\right) \tag{2}$$

where $(1-\alpha)n_i$ is the number (assumed integral) of selective travellers for the ith OD pair, and π_i is the route choice probability distribution for this OD pair alone.

Adjustments to memory length and form of π allow considerable flexibility in the representation traveller learning. Under very general conditions the concatenation of the route flows over m day periods, $\{(\mathbf{x}(t-m+1),\ldots,\mathbf{x}(t)) : t = m, m+1, \ldots\}$, forms an ergodic Markov process (5). The mathematical theory of Markov processes is well understood (see (8), for example), and can help us to understand the properties of this type of traffic assignment model. For example, it is know that (under the aforementioned general conditions) the assignment process will settle down to probabilistic equilibrium as t becomes large, with the route choices described by a stationary probability distribution π^*. Nonetheless, a detailed understanding of short-term and medium-term behaviour of a Markov traffic assignment process defies mathematical analysis for even modestly sized applications. We can, however, learn about such behaviour by simulating the system, a procedure that is facilitated by the Markov structure of the model.

In most applications to date, the conditional route choice probability distribution π has been defined implicitly through a random utility model. In such a situation the probability of a traveller choosing route r is given by

$$\pi_r = \mathrm{P}(V_r = \min\{V_s : s \sim r\}) \tag{3}$$

where V_s is the disutility of route s as perceived by the traveller in question, and $s \sim r$ if and only if routes r and s service the same OD pair. A popular approach of this type is to use logit route choice (see (9), for instance), where the measured disutility (frequently defined as a linear combination of measured route costs over the previous m days) is perturbed by additive Gumbel noise to give individual perceived disutilities. The route choice probabilities can be computed as particular ratios of exponentiated measured costs (8). Nonetheless, it is well known that logit route choices suffers from a lack of IIA (independence to irrelevant alternative) property. An alternative method (without such IIA problems) is probit route choice. In a probit Markov assignment model the vector of measured *link* disutilities on day t is given by

$$\mathbf{V}^*(t) = \sum_{\tau=1}^{m} \beta_\tau \mathbf{k}(t-\tau) \tag{4}$$

where $\beta_1, \ldots, \beta_m$ are weights. The vector of *perceived route* disutilities for a particular traveller on day t is given by

$$\mathbf{V}(t) = A'\{\mathbf{V}^*(t) + \boldsymbol{\epsilon}(t)\} \tag{5}$$

where A' is the path-link incidence matrix (i.e. the transpose of A defined above) and $\boldsymbol{\epsilon}(t)$ is a vector of uncorrelated normal random variables, independent of $\boldsymbol{\epsilon}(t+\tau)$ for $\tau \neq 0$. Note that the variances of the components of $\boldsymbol{\epsilon}$ may depend on the corresponding measured link disutilities. If this dependence is defined so that $\mathrm{var}(\epsilon_i) \propto (V_i^*)^2$ then the stochastic component of the perceived disutility is effectively operating in a multiplicative manner: viz. $V_i^* + \epsilon_i = V_i^*(1+\eta_i)$ where η_i is normally distributed with variance independent of V_i^*. These variances can also be allowed to vary between different groups, permitting selective travellers at day t to be divided into low variance and high variance groups, for example.

While a probit implementation of Markov assignment is attractive from a theoretical standpoint, there are practical difficulties with this approach. The crux of the problem is that the probabilities in equation 3 cannot be derived mathematically in closed form when the disutilities are given by equations 4 and 5. In principle these probabilities can be estimated with an arbitrary degree of precision using sufficient many Monte Carlo simulations. This process requires generation of the random vectors $\boldsymbol{\epsilon}(s)$ $(s = t - m, \ldots, t - 1)$ (from equation 5) to be generated from the appropriate multivariate normal distributions to produce a realized set of perceived disutilities. A single traveller is then assigned to the route with minimum disutility using a standard shortest path algorithm. This process repeated for many iterations for each OD pair. For a given OD pair, the proportions of travellers selecting each route provides estimates of the corresponding route choice probabilities.

Accurate estimation of route choice probabilities by Monte Carlo simulation is computationally intensive. If we wish to estimate a route choice probability whose true value is 0.1, then about 100 simulated route choices (each requiring a shortest path algorithm to be employed) are required to be 95% confident of keeping the relative error below 20%. Around 1500 simulations are required for 95% confidence of obtaining a relative error of 5% or less. Now, for many intents and purposes the route choice probabilities are not of primary interest in themselves – they are required only to allow the evolution of a Markov assignment process to be simulated by sampling according to equation 2 from day-to-day. When this is the case the computational burden can be reduced, particularly in applications where the number of selective travellers for many OD pairs (during the time period of interest) is relatively small (tens of trips, for perhaps). For such OD pairs there is no need to estimate the route choice probabilities with precision, since all the travellers may be assigned by selecting the route with minimum simulated perceived disutility. Not only does this cut down on the number of Monte Carlo iterations required, it also produces an assignment pattern sampled from the exact conditional route choice distribution (rather than some estimate of it). For an OD pair with heavy demand we can still make some use of this idea, using 200 (say) Monte Carlo iterations to perform two tasks simultaneously: (i) assignment of the first 200 travellers, and (ii) estimation of the route choice probabilities for assignment of the remaining travellers.

DEVELOPMENT OF SOFTWARE

We are currently developing a software package MARTS (*M*arkov *A*ssignment for *R*oad *T*raffic *S*ystems) to implement the Markov assignment process with probit route choice described above. This software is written in R, a high level statistical freeware language that is a close relation to the state-of-the-art commercial system, S-Plus; see (10). R is available for (free) download from the Web site `http://www.cran.r-project.org`. The choice of R as the development language was guided by a number issues, including its free availability and the ease with which sampling routines can be constructed in this language. Furthermore, as a statistical package, R provides an excellent environment for analysing the simulation experiments conducted on Markov assignment processes. A disadvantage of using R is that algorithms written in this language run relatively slowly in comparison to analogous algorithms written in lower level languages such as C or Fortran. This is a major issue because of the need for large numbers of shortest path algorithms in order to simulate the evolution of our assignment process. We have overcome this problem to a great extent by dynamically loading into R an efficient shortest path algorithm coded in C. Examples of MARTS output are presented in the next section where we use this package to investigate issues associated with the provision of pre-trip information.

APPLICATION TO PRE-TRIP INFORMATION

Day-to-day assignment models can be of considerable use for the quantitative study of the macroscopic effects of introducing pre-trip information into a network. The explicit manner in which traveller learning and route choice are represented in our model allows the impact of pre-trip information to be examined under a variety of assumptions about travellers' use of such information. For instance, we can assess the impact of pre-trip information under different levels of traveller inertia by suitably adjusting α (the proportion of travellers acting habitually on each day). We can also model the effect of different levels of quality in pre-trip information by altering the covariance matrix of $\boldsymbol{\epsilon}$. For example, in a system with high quality pre-trip information can be represented by having making the elements of this matrix relative small (so that all travellers have a rather accurate knowledge of past travel costs).

We illustrate the use of MARTS for studying the impact of pre-trip information using a portion of the road network from the UK city of Leicester. Specifically, we will look at a subnetwork designated 'region R' for the purposes of traffic management by Leicestershire City Council. This region lies to the south (and slightly east) of Leicester city center. An abstraction of the network in this region is given in Figure 1. In reality the network is orientated so that London Road (a major arterial) runs south-east (node 5) to north-west (node 1). The nodes 1, 5, 10, 12, 13, 14 are major origins and destinations of traffic flow – an OD matrix for this network is given in (12). Link cost functions for this network were quadratic BPR type (13), calibrated by capacity and free-flow speed.

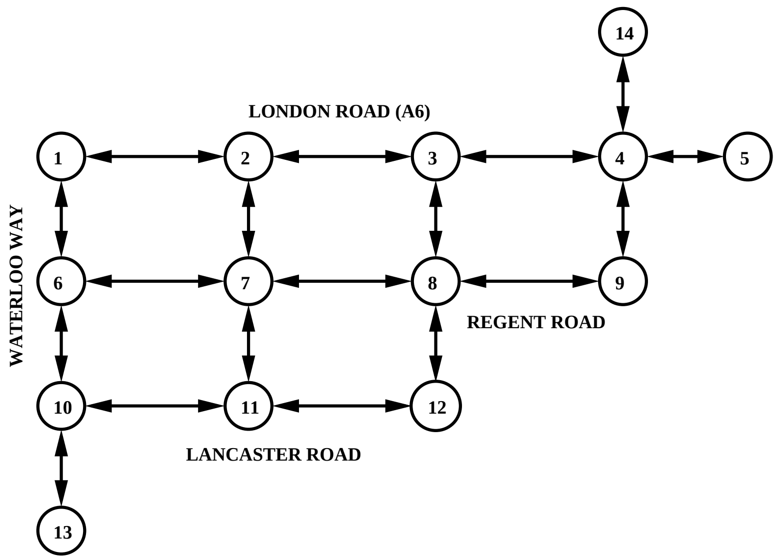

Figure 1: Abstraction of road network in region 'R' of the UK city of Leicester.

We consider the day-to-day evolution of traffic flows over the network under a number of scenarios, as outlined in Table 1. In these scenarios we vary the assignment parameters $1 - \alpha$ (proportion of selective travellers), m (memory length) and the covariance matrix of $\boldsymbol{\epsilon}$. (With regard to this covariance matrix, we assume for simplicity a constant coefficient of variation, σ/μ, for all perceived link costs.) The resulting systems can be summarised in terms of the quality of pre-trip information (high if the link cost 'error' is small, and low if this 'error' is large) and the volatility of the travelling population. In the very volatile systems that we consider all travellers are selective, and base their route choice on information about traffic flows on the previous day alone. In the low volatility systems, most (80%) of travellers are habitual on each day, and those that are selective use travel information based upon the last 5 days. Moderately volatile systems combine either short memory with low selectivity, or long memory with 100% selectivity.

Scenario	description	$1-\alpha$	σ/μ	m
A	Poor information, high volatility	100%	20%	1
B	Good information, high volatility	100%	1%	1
C	Poor information, low volatility	20%	20%	5
D	Good information, low volatility	20%	1%	5
E	Good information, moderate volatility I	100%	1%	5
F	Good information, moderate volatility II	20%	1%	1

Table 1: Simulation scenarios for investigating the effects of pre-trip information. The parameter $1-\alpha$ is the proportion of selective travellers on each day; σ/μ denotes the coefficient of variation in perceived link costs in the probit route choice model; and m is the effective length of memory, in days.

There are many aspects of the simulation results that we could look at for each scenario. In the interests of brevity we look only at total travel cost on network. This cost is plotted day-by-day for each scenario in Figure 2.

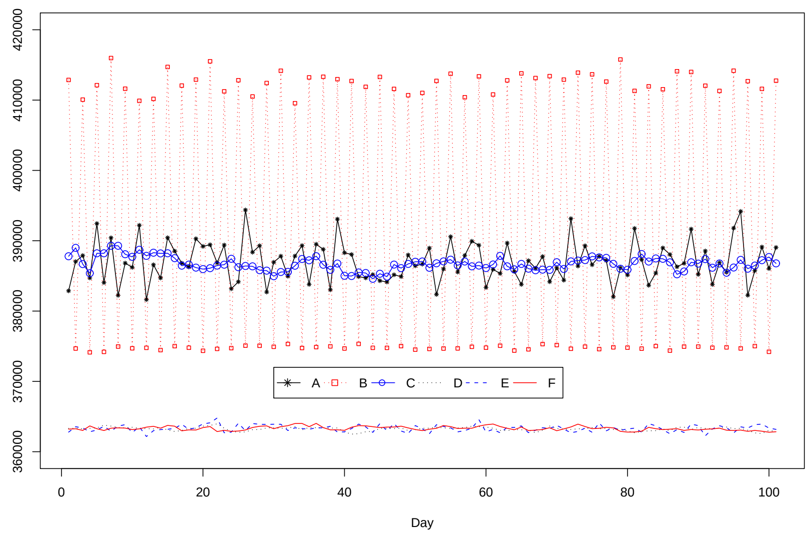

Figure 2: Total cost on network day-by-day for six simulation scenarios. The legend identifies the scenario.

As one might expect, the scenarios with relatively small mean and variance in total cost – D, E and F – all incorporate the provision of high quality pre-trip information to travellers. When the information is of poor quality (scenarios A and C) the mean cost is high, with high variance observed in the volatile scenario A. Interestingly the worst network performance (both in terms of mean and variance of total cost) occurs under scenario B where high quality information is provided. However, the travellers in this case are highly volatile, reacting *en masse* to news about the most recent traffic flows only. As a result, routes that were cheap on day t become heavily over-used on day $t+1$, resulting in congestion and delays. This type of problem does not occur if traveller inertia is high (cf. scenario F) or if travellers tend to balance out travel information from several days (cf. scenario E).

In an extension of this experiment we consider a situation in which road works significantly reduce the capacity of a segment of London Road from day $t = 101$ onwards. (The facility to change network link characteristics at user-specified time-points is available in the current version of MARTS.) We concentrate on the scenarios with 80% of travellers acting habitually on each day i.e. scenarios C, D and F only. In each case the percentage of travellers acting habitually is cut to 50% on the day of the road closure, and then gradually falls back to 20% over the following 4 days (in an attempt to more realistically represent increased traveller reactivity in the face of significant changes to the road system). The results (again in terms of total cost on network) are displayed in Figure 3. As expected, the total cost increases significantly under all scenarios after the introduction of the road works. In all cases there is a spike in the cost plot when the road works are introduced. When high quality information is provided (scenarios D and F) the system reaches a new stochastic equilibrium very swiftly. In the low quality information scenario C, there is evidence of a somewhat slower movement towards a new stationary state.

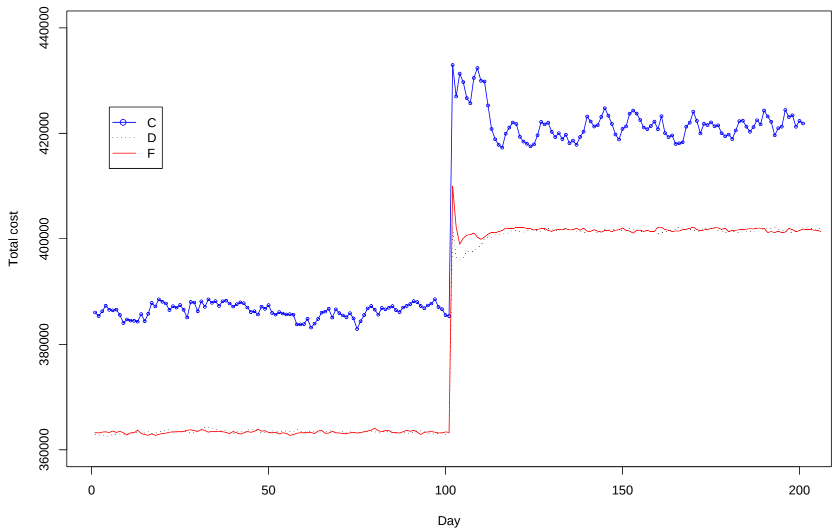

Figure 3: Total cost on network day-by-day for three simulation scenarios, where road works begin on London Road on day 101. The legend identifies the scenario.

CONCLUSIONS

In this paper we have described a new type of Markov traffic assignment model. This model is implemented in a software package MARTS that is currently under development. MARTS allows considerable flexibility in representing the manner in which travellers select routes based upon past experience and pre-trip information. The capabilities of MARTS were illustrated using simulation experiments to investigate network operation under a variety of scenarios. While we recognise that these experiments are rather idealized (and that our conclusions should hence be interpreted with caution) it is interesting to note that the effect of high quality pre-trip information in highly volatile systems may not be entirely beneficial.

The use of efficient algorithms and coding in MARTS allows us to simulate the evolution of modestly size networks over hundreds of days. Nonetheless, probit Markov assignment is not currently feasible for large networks and long simulation runs. We can perhaps make progress here by using regression techniques to estimate route choice probabilities for later days based on simulated assignment flows on earlier days. Our work in this area is ongoing.

REFERENCES

(1) Wardrop, J.G. (1952) Some theoretical aspects of road traffic research. *Proceedings of the Institute of Civil Engineering, Part II, 1*, 325-378.

(2) Watling, D.P. and Van Vuren, T. (1993) The modelling of dynamic route guidance systems. *Transportation Research Part C, 1*, 159-182.

(3) Polak, J.W. and Hazelton, M.L. (1998) The influence of alternative traveller learning mechanisms on the dynamics of transport systems. In *European Transport Conference, Proceedings of Seminar D: Transport Planning and Methods*, 83-95.

(4) Hazelton, M.L. and Watling, D.P. (2001) Computing moments for stochastic traffic assignment models. Under revision for *Transportation Science*.

(5) Cascetta, E. (1989) A stochastic process approach to the analysis of temporal dynamics in transportation networks. *Transportation Research Part B, 23*, 1-17.

(6) Cantarella, G.E. (1993) Day-to-day dynamics in transportation networks: stability and limits of equilibrium in a two-link network. *Sistemi Urbani, 1*, 27-50.

(7) Watling, D.P. and Hazelton, M.L. (2001) The dynamics and equilibria of day-to-day assignment models. To appear in *Networks and Spatial Economics*.

(8) Gamerman, D. (1997) *Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference*. Chapman & Hall: London.

(9) Thomas, R. (1991) *Traffic Assignment Techniques*. Averbury Technical: Aldershot.

(10) Venables, W.N and Ripley, B.D. (1999) *Modern Applied Statistics with S-Plus* (3rd ed.). Springer: New York.

(11) Bell, M. (1993) A study of changes in traffic patterns following traffic management schemes. Technical Report TP93023, Nottingham University Transport Research Group: Nottingham, UK.

(12) Hazelton, M.L. (2001) Estimation of Origin-Destination Trip Rates in Leicester. To appear in *Applied Statistics*.

(13) Bureau of Public Roads (1964) *Traffic Assignment Manual*. US Department of Commerce: Washington DC.